# **Multi-Qubit Quantum Systems**

## **Tensor (Direct) Product of States**

A multi-qubit system combines states of individual qubits using the tensor product (or direct product). For two qubits:

* If the first qubit is in state |ϕ⟩ = α|0⟩ + β|1⟩ and the second qubit in state |ψ⟩ = α'|0⟩ + β'|1⟩, the combined system is:  
  |ϕψ⟩ = αα'|00⟩ + αβ'|01⟩ + βα'|10⟩ + ββ'|11⟩
* Basis states: For 2 qubits, there are 4 orthonormal basis states: |00⟩, |01⟩, |10⟩, |11⟩.

## **Measurement in Multi-Qubit Systems**

Measurement collapses the state to one of the basis states with probabilities determined by the squared amplitudes.

Example: For the state: |ψ⟩ = α₁|00⟩ + α₂|01⟩ + α₃|10⟩ + α₄|11⟩:

* Probability of measuring |00⟩: |α₁|²
* Probability of measuring |01⟩: |α₂|²

## **Entanglement**

Entanglement occurs when a multi-qubit state cannot be written as a direct product of single-qubit states.

Example: The Bell State is entangled:

* |Bell⟩ = 1/√2 (|00⟩ + |11⟩)

Partial Measurement Example:

* Measuring one qubit in an entangled state (e.g., |Bell⟩) collapses both qubits to correlated states.

## **Probability in Combined Systems**

Given two qubits in superposition:  
|ϕ⟩ = (1/√2)(|0⟩ + |1⟩), |ψ⟩ = (√3/2)|0⟩ + (1/2)|1⟩

The combined state becomes:  
|ϕψ⟩ = 1/√2 ⊗ (√3/2)|00⟩ + 1/2|01⟩ + ...

Measurement collapses states with probabilities equal to the squared amplitudes of coefficients.

## **Partial Measurement**

In entangled systems, measuring one qubit affects the other qubit's state.  
Example: For the Bell State |ψ⟩ = 1/√2 (|00⟩ + |11⟩):

* If qubit 1 is measured as |0⟩, qubit 2 collapses to |0⟩.
* If qubit 1 is measured as |1⟩, qubit 2 collapses to |1⟩.

## **More Than Two Qubits**

For a 3-qubit system, there are 8 orthonormal basis states (e.g., |000⟩, |001⟩, ..., |111⟩).  
Entanglement extends to systems with multiple qubits, and not all states can be expressed as direct products.